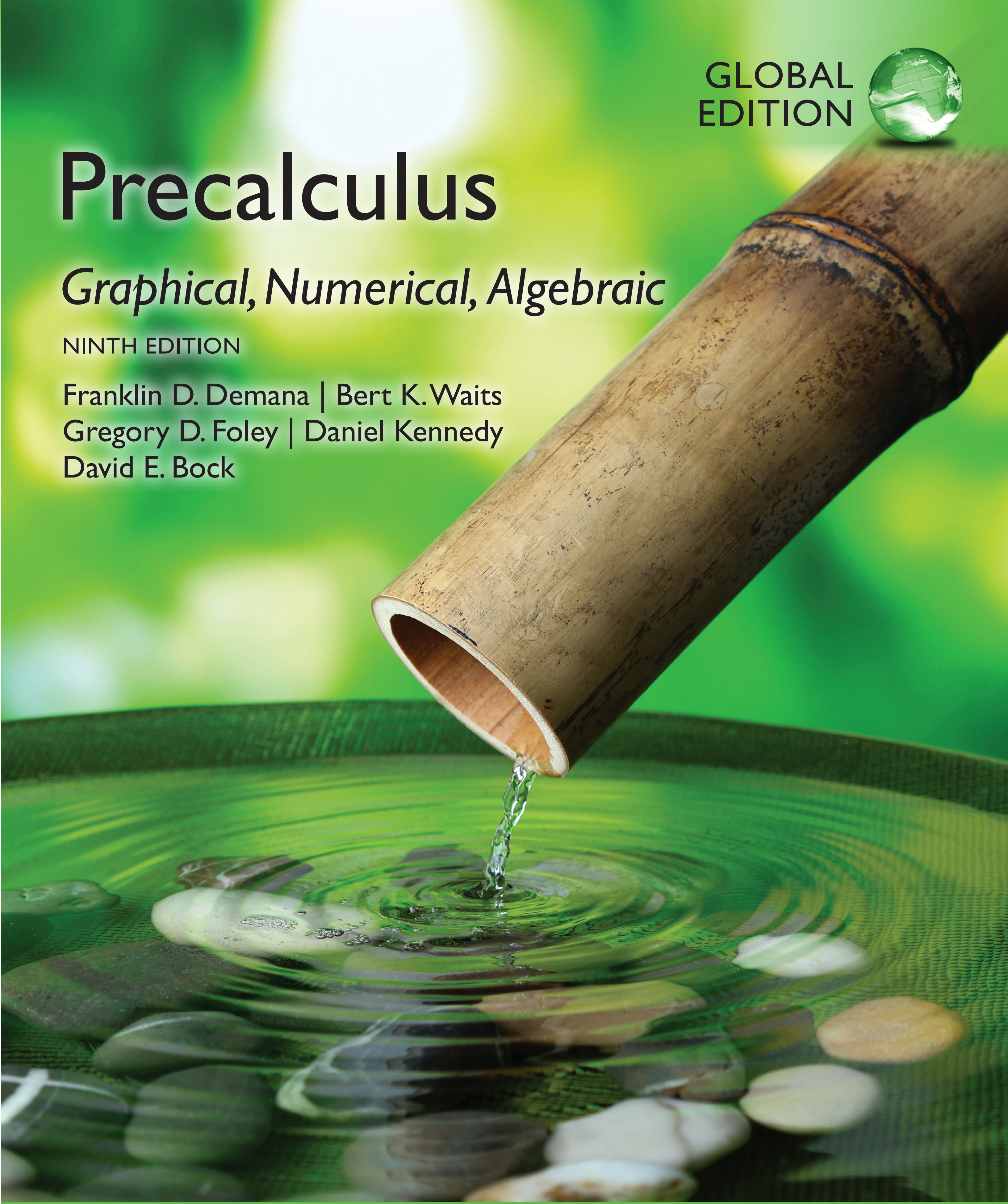
GLOBAL EDITION

Precalculus

Graphical, Numerical, Algebraic

NINTH EDITION

Franklin D. Demana | Bert K. Waits
Gregory D. Foley | Daniel Kennedy
David E. Bock



ALWAYS LEARNING

PEARSON

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Ninth Edition
Global Edition

Franklin D. Demana
The Ohio State University

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Gregory D. Foley
Ohio University

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Baylor School

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FOREWORD

We are proud of the fact that earlier editions of *Precalculus: Graphical, Numerical, Algebraic* were among the first to recognize the potential of hand-held graphers for helping students understand function behavior. The power of visualization eventually transformed the teaching and learning of calculus on the college level and in the AP® program, then led to reforms in the high school curriculum articulated in the NCTM *Principles and Standards for School Mathematics* and now in the Common Core State Standards. All along the way, this textbook has kept current with the best practices while continuing to pioneer new ideas in exploration and pedagogy that enhance student learning (for example, the study of function behavior based on the Twelve Basic Functions, an idea that has gained widespread acceptance in the textbook world). For those students continuing to a calculus course, this precalculus textbook concludes with a chapter that prepares students for the two central themes of calculus: instantaneous rate of change and continuous accumulation. This intuitively appealing preview of calculus is both more useful and more reasonable than the traditional, unmotivated foray into the computation of limits, and it is more in keeping with the stated goals and objectives of the AP courses and their emphasis on depth of knowledge.

Recognizing that precalculus is a capstone course for many students, we include *quantitative literacy* topics such as probability, statistics, and the mathematics of finance and integrate the use of data and modeling throughout the text. Our goal is to provide students with the critical-thinking skills and mathematical know-how needed to succeed in college, career, or any endeavor.

Continuing in the spirit of the eight earlier editions, we have integrated graphing technology throughout the course, not as an additional topic but as an essential tool for both mathematical discovery and effective problem solving. Graphing technology enables students to study a full catalog of basic functions at the beginning of the course, thereby giving them insights into function properties that are not seen in many books until later chapters. By connecting the algebra of functions to the visualization of their graphs, we are even able to introduce students to parametric equations, piecewise-defined functions, limit notation, and an intuitive understanding of continuity as early as Chapter 1. However, the advances in technology and increased familiarity with calculators have blurred some of the distinctions between solving problems and supporting solutions that we had once assumed to be apparent. Therefore, we ask that some exercises be solved without calculators. (See the “Technology and Exercises” section.)

Once students are comfortable with the language of functions, the text guides them through a more traditional exploration of twelve basic functions and their algebraic properties, always reinforcing the connections among their algebraic, graphical, and numerical representations. This book uses a consistent approach to modeling, emphasizing the use of particular types of functions to model behavior in the real world. Modeling is a fundamental aspect of our problem-solving process that is introduced in Section 1.1 and used throughout the book. The text has a wealth of data and range of applications to illustrate how mathematics and statistics connect to every facet of modern life.

This textbook has faithfully incorporated not only the teaching strategies that have made *Calculus: Graphical, Numerical, Algebraic* so popular, but also some of the strategies from the popular Pearson high school algebra series, and thus has produced a seamless pedagogical transition from prealgebra through calculus for students. Although this book can certainly be appreciated on its own merits, teachers who seek coherence and vertical alignment in their mathematics sequence might consider this pedagogical approach to be an additional asset of *Precalculus: Graphical, Numerical, Algebraic.*

This textbook is written to address current and emerging state curriculum standards. In particular, we embrace NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* and its emphasis on the importance of helping students make sense of and reason using mathematics. The NCTM's *Principles and Standards for School Mathematics* identified five "Process Standards" that should be fundamental in mathematics education. The first of these standards was Problem Solving. Since then, the emphasis on problem solving has continued to grow, to the point that it is now integral to the instructional process in many mathematics classrooms. When the Common Core State Standards in Mathematics detailed eight "Standards for Mathematical Practice" that should be fundamental in mathematics education, again the first of these addressed problem solving. Individual states have also released their own standards over the years, and problem solving is invariably front and center as a fundamental objective. Problem solving, reasoning, sense making, and the related processes and practices of mathematics are central to the approach we use in *Precalculus: Graphical, Numerical, Algebraic.*

With this new edition we embrace the growing importance and wide applicability of Statistics. Because Statistics is increasingly used in college coursework, the workplace, and everyday life, we have added a new Chapter 10 to help students see that statistical analysis is an investigative process that turns loosely formed ideas into scientific studies. Our five sections on data analysis, probability, and statistical literacy are aligned with the *GAISE* Report published by the American Statistical Association, the College Board's AP® Statistics curriculum, and the Common Core State Standards. Chapter 10 is not intended as a course in statistics but rather as an introduction to set the stage for possible further study.

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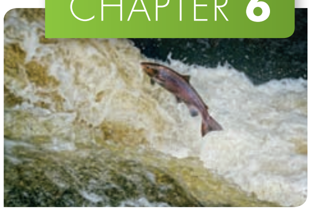

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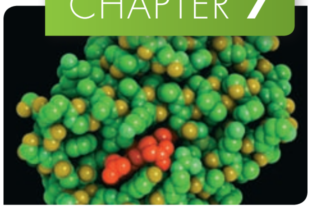

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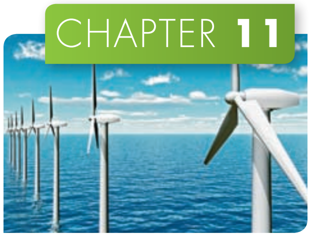

An Introduction to Calculus: Limits, Derivatives, and Integrals

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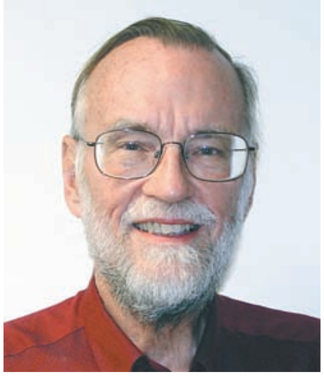
Dr. Demana coauthored *Calculus: Graphical, Numerical, Algebraic; Essential Algebra: A Calculator Approach; Transition to College Mathematics; College Algebra and Trigonometry: A Graphing Approach; College Algebra: A Graphing Approach; Precalculus: Functions and Graphs;* and *Intermediate Algebra: A Graphing Approach.*



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Bert Waits received his Ph.D. from The Ohio State University and is currently Professor Emeritus of Mathematics there. Dr. Waits is cofounder of the international Teachers Teaching with Technology (T^3) professional development program, and has been codirector or principal investigator on several large National Science Foundation projects. Dr. Waits has published articles in more than 70 nationally recognized professional journals. He frequently has given invited lectures, workshops, and minicourses at national meetings of the MAA and the National Council of Teachers of Mathematics (NCTM) on how to use computer technology to enhance the teaching and learning of mathematics. Dr. Waits is co-recipient of the 1997 Glenn Gilbert National Leadership Award presented by the National Council of Supervisors of Mathematics, and is the cofounder (with Frank Demana) of the ICTCM. He is also co-recipient of the 1998 Christofferson-Fawcett Mathematics Education Award presented by the Ohio Council of Teachers of Mathematics. Dr. Waits was one of the six authors of the high school portion of the groundbreaking 1989 *NCTM Standards.*

Dr. Waits coauthored *Calculus: Graphical, Numerical, Algebraic; College Algebra and Trigonometry: A Graphing Approach; College Algebra: A Graphing Approach; Precalculus: Functions and Graphs;* and *Intermediate Algebra: A Graphing Approach*.

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Dr. Foley coauthored *Precalculus: A Graphing Approach*; *Precalculus: Functions and Graphs*; and *Advanced Quantitative Reasoning: Mathematics for the World Around Us*.

Daniel Kennedy

Dan Kennedy received his undergraduate degree from the College of the Holy Cross and his master's degree and Ph.D. in mathematics from the University of North Carolina at Chapel Hill. Since 1973 he has taught mathematics at the Baylor School in Chattanooga, Tennessee, where he holds the Cartter Lupton Distinguished Professorship. Dr. Kennedy joined the Advanced Placement® Calculus Test Development Committee in 1986, then in 1990 became the first high school teacher in 35 years to chair that committee. It was during his tenure as chair that the program moved to require graphing calculators and laid the early groundwork for the 1998 reform of the Advanced Placement Calculus curriculum. The author of the 1997 *Teacher's Guide—AP® Calculus*, Dr. Kennedy has conducted more than 50 workshops and institutes for high school calculus teachers. His articles on mathematics teaching have appeared in the *Mathematics Teacher* and the *American Mathematical Monthly*, and he is a frequent speaker on education reform at professional and civic meetings. Dr. Kennedy was named a Tandy Technology Scholar in 1992 and a Presidential Award winner in 1995.

Dr. Kennedy coauthored *Calculus: Graphical, Numerical, Algebraic; Prentice Hall Algebra I; Prentice Hall Geometry;* and *Prentice Hall Algebra 2*.

David E. Bock

Dave Bock holds degrees from the University at Albany (NY) in mathematics (B.A.) and statistics/education (M.S.). Mr. Bock taught mathematics at Ithaca High School for 35 years, including both BC Calculus and AP Statistics. He also taught Statistics at Tompkins-Cortland Community College, Ithaca College, and Cornell University, where he recently served as K–12 Education and Outreach Coordinator and Senior Lecturer for the Mathematics Department. Mr. Bock serves as a Statistics consultant to the College Board, leading numerous workshops and institutes for AP Statistics teachers. He has been a reader for the AP Calculus exam and both a reader and a table leader for the AP Statistics exam. During his career Mr. Bock won numerous teaching awards, including the MAA's Edyth May Sliffe Award for Distinguished High School Mathematics Teaching (twice) and Cornell University's Outstanding Educator Award (three times), and was also a finalist for New York State Teacher of the Year.

Mr. Bock coauthored the AP Statistics textbook *Stats: Modeling the World*, the non-AP text *Stats in Your World*, Barron's *AP Calculus* review book, and Barron's *AP Calculus Flash Cards*.

PREFACE

Our Approach

The Rule of Four—A Balanced Approach

A principal feature of this text is the balance among the algebraic, numerical, graphical, and verbal methods of representing problems: the rule of four. For instance, we obtain solutions algebraically when that is the most appropriate technique to use, and we obtain solutions graphically or numerically when algebra is difficult to use. We urge students to solve problems by one method and then support or confirm their solutions by using another method. We believe that students must learn the value of each of these methods or representations and must learn to choose the one most appropriate for solving the particular problem under consideration. This approach reinforces the idea that to understand a problem fully, students need to understand it algebraically as well as graphically and numerically.

Problem-Solving Approach

Systematic problem solving is emphasized in the examples throughout the text, using the following variation of Polya's problem-solving process:

- *understand* the problem,
- *develop* a mathematical model,
- *solve* the mathematical model and support or confirm the solutions, and
- *interpret* the solution.

Students are encouraged to use this process throughout the text.

Twelve Basic Functions

Twelve basic functions are emphasized throughout the book as a major theme and focus. These functions are

- The Identity Function
- The Squaring Function
- The Cubing Function
- The Reciprocal Function
- The Square Root Function
- The Exponential Function
- The Natural Logarithm Function
- The Sine Function
- The Cosine Function
- The Absolute Value Function
- The Greatest Integer Function
- The Logistic Function

One of the most distinctive features of this textbook is that it introduces students to the full vocabulary of functions early in the course. Students meet the twelve basic functions graphically in Chapter 1 and are able to compare and contrast them as they learn about concepts like domain, range, symmetry, continuity, end behavior, asymptotes, extrema, and even periodicity—concepts that are difficult to appreciate when the only examples a teacher can refer to are polynomials. With this book, students are able to characterize functions by their behavior within the first month of classes. Once students have a comfortable understanding of functions in general, the rest of the course consists of studying the various types of functions in greater depth, particularly with respect to their algebraic properties and modeling applications.

These functions are used to develop the fundamental analytic skills that are needed in calculus and advanced mathematics courses. A complete gallery of basic functions is included in Appendix C and inside the back cover of the book for easy reference.

Applications and Real Data

The majority of the applications in the text are based on real data from cited sources, and their presentations are self-contained. As they work through the applications, students are exposed to functions as mechanisms for modeling real-life problems. They learn to analyze and model data, represent data graphically, interpret from graphs, and fit curves. Additionally, the tabular representation of data presented in this text highlights the concept that a function is a correspondence between numerical variables. This helps students build the connection between numerical quantities and graphs and recognize the importance of a full graphical, numerical, and algebraic understanding of a problem. For a complete listing of applications, please see the Applications Index on page 977.

Technology and Exercises

The authors of this textbook have encouraged the use of technology in mathematics education for three decades. Our approach to problem solving (pages 94–95) distinguishes between **solving** the problem and **supporting** or **confirming** the solution, and how technology figures into each of those processes.

We have come to realize, however, that advances in technology and increased familiarity with calculators have gradually blurred some of the distinctions between solving and supporting that we had once assumed to be apparent. We do not want to retreat in any way from our support of modern technology, but we feel that the time has come to provide more guidance about the intent of the various exercises in our textbook.

Therefore, as a service to teachers and students alike, exercises in this textbook that **should be solved without calculators** are identified with gray ovals around the exercise numbers. These usually are exercises that demonstrate how various functions behave algebraically or how algebraic representations reflect graphical behavior and vice versa. Application problems usually have no restrictions, in keeping with our emphasis on **modeling** and on bringing **all representations** to bear when confronting real-world problems.

Incidentally, we continue to encourage the use of calculators to **support** answers graphically or numerically after the problems have been solved with pencil and paper. Any time students can make connections among the graphical, analytical, and numerical representations, they are doing good mathematics.

As a final note, we will freely admit that different teachers use our textbook in different ways, and some will probably override our no-calculator recommendations to fit with their pedagogical strategies. In the end, the teachers know what is best for their students, and we are just here to help.

Content Changes to This Edition

Mindful of the need to keep the applications of mathematics relevant to our students, we have changed many of the examples and exercises throughout the book to include the most current data available to us at the time of publication. Additionally, calculator screens were updated to conform to the enhanced capabilities of modern graphers. We have also added more student and teacher notes. The importance of statistics and probability for college, career, and everyday life has grown to the point that we now have a separate chapter titled Statistics and Probability. The Common Core Edition is built upon the prior editions of this textbook.

In **Chapter P** the use of the point-slope form of a line has been integrated into the solution of more examples. In **Chapter 1** references to calculator regression models were reworded to avoid giving the wrong signals about how statisticians actually operate. The discussion of linear correlation was revised in **Chapter 2** to complement this edition's more extensive treatment of Statistics. The section on financial mathematics in **Chapter 3** was updated, and simple interest was included. Additionally, a predator-prey application was added.

In **Chapter 6**, several significant textual changes have been made in order to tie the topics of this chapter (vectors, parametric equations, and polar graphing) more directly to the topics in the preceding chapters, particularly the unifying concepts of functions and their graphs in the Cartesian plane. The material on partial fractions has been incorporated into Section 7.3 to streamline **Chapter 7**.

Within **Chapter 8** the treatment of conic sections has been changed to emphasize that they are extensions of previously studied topics, even if their graphs do not pass the vertical line test. Explorations have been added to allow students to make the connections with earlier topics; for example, rotation of axes is introduced by prompting students to treat one of their twelve basic functions (the reciprocal function) as a hyperbola and find its vertices and foci.

The new **Chapter 10** expands the discussion of Statistics and probability. Section 10.1 opens the chapter with a discussion of basic probability concepts, including sample spaces, determining probabilities of compound events, Venn diagrams, tree diagrams, and conditional probability. Section 10.2 examines the creation and interpretation of graphical displays of data, including pie charts and bar charts of categorical data, stemplots and histograms for quantitative data, and time plots. Section 10.3 presents numerical summaries of center and spread for describing and comparing distributions, including the five-number summary, mean, and standard deviation, and introduces both boxplots and the Normal curve. Section 10.4 expands the discussion of probability to include random variables and probability models, including expected value, binomial probabilities, and Normal probabilities, and links these models to data and decision making by introducing the concept of statistical significance. Section 10.5 closes the chapter with a broad look at statistical literacy, the design of statistical studies, the important role of randomness, and the use of simulations to estimate probabilities and assess statistical significance. Throughout the chapter the emphasis is on proper statistical terminology and practice, attention to applications, and statistical thinking.

Features

Chapter Openers include a general description of an application that can be solved with the concepts learned in the chapter. The application is revisited later in the chapter via a specific problem that is solved.

A **Chapter Overview** begins each chapter to give students a sense of what they are going to learn. This overview provides a roadmap of the chapter, as well as tells how the topics in the chapter are connected under one big idea. It is always helpful to remember that mathematics isn't modular, but interconnected, and that the skills and concepts learned throughout the course build on one another to help students understand more complicated processes and relationships. Similarly, the **What you'll learn about . . . and why** feature presents the big ideas in each section and explains their purpose.

Throughout the book, **Vocabulary** is highlighted in yellow for easy reference. Additionally, **Properties**, **Definitions**, and **Theorems** are boxed in blue, and **Procedures** in purple, so that they can be easily found. The **Web/Real Data** icon marks the examples and exercises that use real cited data.

Each example ends with a suggestion to **Now Try** a related exercise. Working the suggested exercise is an easy way for students to check their comprehension of the material while reading each section.

Explorations appear throughout the text and provide students with the perfect opportunity to become active learners and to discover mathematics on their own. This will help hone critical-thinking and problem-solving skills. Some are technology-based and others involve exploring mathematical ideas and connections.

Margin Notes and Tips on various topics appear throughout the text. *Tips* offer practical advice on using the grapher to obtain the best, most accurate results. *Margin notes* include historical information and hints about examples, and provide additional insight to help students avoid common pitfalls and errors.

The **Looking Ahead to Calculus** icon is found throughout the text next to many examples and topics to point out concepts that students will encounter again in calculus. Ideas that foreshadow calculus, such as limits, maximum and minimum, asymptotes, and continuity, are highlighted. Some calculus notation and language are introduced in the early chapters and used throughout the text to establish familiarity.

The **Chapter Review** material at the end of each chapter consists of sections dedicated to helping students review the chapter concepts. **Key Ideas** are broken into parts: Properties, Theorems, and Formulas; Procedures; and Gallery of Functions. The **Review Exercises** represent the full range of exercises covered in the chapter and give additional practice with the ideas developed in the chapter. The exercises with red numbers indicate problems that would make up a good chapter test. **Chapter Projects** conclude each chapter and require students to analyze data. They can be assigned as either individual or group work. Each project expands upon concepts and ideas taught in the chapter, and many projects refer to the Web for further investigation of real data.

Exercise Sets

Each exercise set begins with a **Quick Review** to help students review skills needed in the exercise set and references others sections students can go to for help. Some exercises are designed to be solved *without* a *calculator*; the numbers of these exercises are printed within a gray oval. Students are urged to **support** the answers to these (and all) exercises graphically or numerically, but only after they have solved them with pencil and paper.

There are over 6000 exercises, including 720 Quick Review Exercises. The section exercises have been carefully graded from routine to challenging. The following types of skills are tested in each exercise set:

- Algebraic and analytic manipulation
- Connecting algebra to geometry
- Interpretation of graphs
- Graphical and numerical representations of functions
- Data analysis

Also included in the exercise sets are thought-provoking exercises:

- **Standardized Test Questions** include two true-false problems with justifications and four multiple-choice questions.
- **Explorations** are opportunities for students to discover mathematics on their own or in groups. These exercises often require the use of critical thinking to explore the ideas.

- **Writing to Learn** exercises give students practice at communicating about mathematics and opportunities to demonstrate understanding of important ideas.
- **Group Activity** exercises ask students to work on the problems in groups or solve them as individual or group projects.
- **Extending the Ideas** exercises go beyond what is presented in the textbook. These exercises are challenging extensions of the book's material.

This variety of exercises provides sufficient flexibility to emphasize the skills most needed for each student or class.

Technology Resources

The following supplements are available for purchase:

MyMathLab® Online Course (optional, for purchase only)—access code required

MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And it comes from a **trusted partner** with educational expertise and an eye on the future. To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson Sales Representative. In this **MyMathLab®** course, you have access to the most cutting-edge, innovative study solutions proven to increase students' success.

Additional Teacher Resources

Most of the teacher supplements and resources available for this text are available electronically for download at the Instructor Resource Center (IRC). Please go to www.pearsonglobaleditions.com/demana *and select "we need IRC (Instructor Resource Access)." You will be required to complete a one-time registration subject to verification before being emailed access information for download materials. Once logged into the IRC, enter the Student Edition ISBN in the* ***Search our Catalog*** *box to locate your resources.*

The following supplements are available to qualified adopters:

Online Solutions Manual (Download Only)

Provides complete solutions to all exercises, including Quick Reviews, Exercises, Explorations, and Chapter Reviews.

Online Resource Manual (Download Only)

Provides Major Concepts Review, Group Activity Worksheets, Sample Chapter Tests, Standardized Test Preparation Questions, Contest Problems.

Online Tests and Quizzes (Download Only)

Provides two parallel tests per chapter, two quizzes for every three to four sections, two parallel midterm tests covering Chapters P–5, and two parallel end-of-year tests, covering Chapters 6–10.

TestGen® (Download Only)

TestGen enables teachers to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing teachers to create multiple but equivalent versions of the same question or test with the click of a button. Teachers can also modify test bank questions or add new questions. Tests can be printed or administered online.

PowerPoint Slides (Download Only)

Features presentations written and designed specifically for this text, including figures, alternate examples, definitions, and key concepts.

Web Site

Our Web site, www.pearsonglobaleditions.com/demana, provides dynamic resources for teachers and students. Some of the resources include TI graphing calculator downloads, online quizzing, teaching tips, study tips, Explorations, end-of-chapter projects, and more.

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—F. D. D.
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CHAPTER P

Prerequisites

Large distances are measured in *light years*, the distance that light travels in one year. Scientists use the speed of light, which is roughly 299,800 km/sec, to approximate distances within the solar system. See page 59 for examples.

Chapter P Overview

Historically, algebra was used to represent problems with symbols (algebraic models) and solve them by reducing the solution to algebraic manipulation of symbols. This technique is still important today. In addition, graphing calculators are now used to represent problems with graphs (graphical models) and solve them with the numerical and graphical techniques of technology.

We begin with basic properties of real numbers and introduce absolute value, distance formulas, midpoint formulas, and equations of circles. Slope of a line is used to write standard equations for lines, and applications involving linear equations are discussed. The basics of complex numbers are explained. Equations and inequalities are solved using both algebraic and graphical techniques.

P.1 Real Numbers

What you'll learn about

- Representing Real Numbers
- Order and Interval Notation
- Basic Properties of Algebra
- Integer Exponents
- Scientific Notation

... and why

These topics are fundamental in the study of mathematics and science.

Representing Real Numbers

A **real number** is any number that can be written as a decimal. Real numbers are represented by symbols such as $-8, 0, 1.75, 2.333..., 0.\overline{36}, 8/5, \sqrt{3}, \sqrt[3]{16}, e,$ and π.

The set of real numbers contains several important subsets:

The **natural (or counting) numbers**:	$\{1, 2, 3, \ldots\}$
The **whole numbers**:	$\{0, 1, 2, 3, \ldots\}$
The **integers**:	$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Braces { } are used to enclose the **elements**, or **objects**, of a set. The rational numbers are another important subset of the real numbers. A **rational number** is any number that can be written as a ratio a/b of two integers, where $b \neq 0$. We can use **set-builder notation** to define the rational numbers:

$$\left\{\frac{a}{b}\,\middle|\, a, b \text{ are integers, and } b \neq 0\right\}$$

The vertical bar that follows a/b is read "such that."

The decimal form of a rational number either **terminates** like $7/4 = 1.75$, or is **infinitely repeating** like $4/11 = 0.363636... = 0.\overline{36}$. The bar over the 36 indicates the block of digits that repeats. A real number is **irrational** if it is *not* rational. The decimal form of an irrational number is infinitely nonrepeating. For example, $\sqrt{3} = 1.7320508...$ and $\pi = 3.14159265...$.

A real number can be approximated by giving a few of its digits. Sometimes we can find the decimal form of rational numbers with calculators, but not very often.

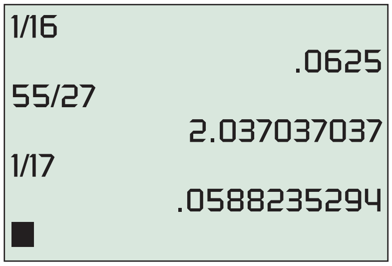


FIGURE P.1 Calculator decimal representations of 1/16, 55/27, and 1/17 with the calculator set in floating decimal mode. (Example 1)

EXAMPLE 1 Examining Decimal Forms of Rational Numbers

Determine the decimal form of 1/16, 55/27, and 1/17.

SOLUTION Figure P.1 suggests that the decimal form of 1/16 terminates and that of 55/27 repeats in blocks of 037.

$$\frac{1}{16} = 0.0625 \quad \text{and} \quad \frac{55}{27} = 2.\overline{037}$$

We cannot predict the *exact* decimal form of 1/17 from Figure P.1; however, we can say that $1/17 \approx 0.0588235294$. The symbol $\approx$ is read "*is approximately equal to*." We can use long division (see Exercise 66) to prove that

$$\frac{1}{17} = 0.\overline{0588235294117647}.$$

Now try Exercise 3.

The real numbers and the points of a line can be matched *one-to-one* to form a **real number line**. We start with a horizontal line and match the real number zero with a point O, the **origin**. **Positive numbers** are assigned to the right of the origin, and **negative numbers** to the left, as shown in Figure P.2.

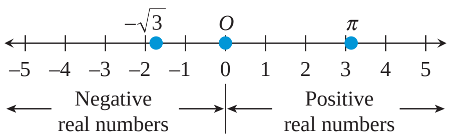


FIGURE P.2 The real number line.

Every real number corresponds to one and only one point on the real number line, and every point on the real number line corresponds to one and only one real number. Between every pair of real numbers on the number line there are infinitely many more real numbers.

The number associated with a point is **the coordinate of the point**. As long as the context is clear, we will follow the standard convention of using the real number for both the name of the point and its coordinate.

Order and Interval Notation

The set of real numbers is **ordered**. This means that we can use inequalities to compare any two real numbers that are not equal and say that one is "less than" or "greater than" the other.

Unordered Systems

Not all number systems are ordered. For example, the complex number system, to be introduced in Section P.6, has no natural ordering.

Order of Real Numbers

Let a and b be any two real numbers.

Symbol	Definition	Read
$a > b$	$a - b$ is positive	a is greater than b
$a < b$	$a - b$ is negative	a is less than b
$a \geq b$	$a - b$ is positive or zero	a is greater than or equal to b
$a \leq b$	$a - b$ is negative or zero	a is less than or equal to b

The symbols $>$, $<$, $\geq$, and $\leq$ are **inequality symbols**.

Opposites and Number Line

$a < 0 \Leftrightarrow -a > 0$

If $a < 0$, then a is to the left of 0 on the real number line, and its opposite, $-a$, is to the right of 0. Thus, $-a > 0$. This logic can be reversed: If $-a > 0$, then $a < 0$.

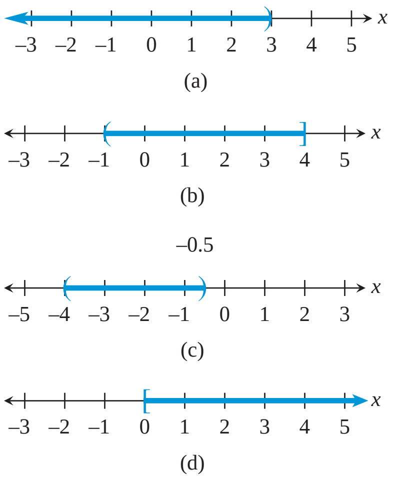


FIGURE P.3 In graphs of inequalities, parentheses correspond to $<$ and $>$ and brackets to $\leq$ and $\geq$. (Examples 2 and 3)

Geometrically, $a > b$ means that a is to the right of b (equivalently b is to the left of a) on the real number line. For example, $6 > 3$ implies that 6 is to the right of 3 on the real number line. Note also that $a > 0$ means that $a - 0$, or simply a, is positive, and $a < 0$ means that a is negative.

We are able to compare any two real numbers because of the following important property of the real numbers.

Trichotomy Property

Let a and b be any two real numbers. Exactly one of the following is true:

$$a < b, \quad a = b, \quad \text{or} \quad a > b$$

Inequalities can be used to describe **intervals** of real numbers, as illustrated in Example 2.

EXAMPLE 2 Interpreting Inequalities

Describe and graph the interval of real numbers for the inequality.

(a) $x < 3$ **(b)** $-1 < x \leq 4$

SOLUTION

(a) The inequality $x < 3$ describes all real numbers less than 3 (Figure P.3a).

(b) The *double inequality* $-1 < x \leq 4$ represents all real numbers between -1 and 4, excluding -1 and including 4 (Figure P.3b). **Now try Exercise 5.**

EXAMPLE 3 Writing Inequalities

Write an interval of real numbers using an inequality and draw its graph.

(a) The real numbers between -4 and -0.5

(b) The real numbers greater than or equal to zero

SOLUTION

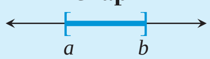
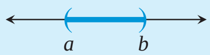
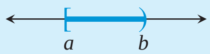
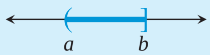
(a) $-4 < x < -0.5$ (Figure P.3c)

(b) $x \geq 0$ (Figure P.3d) **Now try Exercise 15.**

As shown in Example 2, inequalities define *intervals* on the real number line. We often use $[2, 5]$ to describe the *bounded interval* determined by $2 \leq x \leq 5$. This interval is **closed** because it contains its *endpoints* 2 and 5. There are four types of **bounded intervals**.

Bounded Intervals of Real Numbers

Let a and b be real numbers with $a < b$.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, b]$	Closed	$a \leq x \leq b$	
(a, b)	Open	$a < x < b$	
$[a, b)$	Half-open	$a \leq x < b$	
$(a, b]$	Half-open	$a < x \leq b$	

The numbers a and b are the **endpoints** of each interval.

We use the interval notation $(-\infty, \infty)$ to represent the entire set of real numbers. The symbols $-\infty$ (*negative infinity*) and ∞ (*positive infinity*) allow us to use interval notation for unbounded intervals and are *not* real numbers.

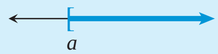
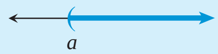
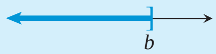
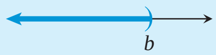
Interval Notation at $\pm\infty$

Because $-\infty$ is *not* a real number, we use $(-\infty, 2)$ instead of $[-\infty, 2)$ to describe $x < 2$. Similarly, we use $[-1, \infty)$ instead of $[-1, \infty]$ to describe $x \geq -1$.

The interval of real numbers determined by the inequality $x < 2$ can be described by the *unbounded interval* $(-\infty, 2)$. This interval is **open** because it does *not* contain its endpoint 2. In addition to $(-\infty, \infty)$, there are four types of **unbounded intervals**.

Unbounded Intervals of Real Numbers

Let a and b be real numbers.

Interval Notation	Interval Type	Inequality Notation	Graph
$[a, \infty)$	Closed	$x \geq a$	
(a, ∞)	Open	$x > a$	
$(-\infty, b]$	Closed	$x \leq b$	
$(-\infty, b)$	Open	$x < b$	

Each of these intervals has exactly one endpoint, namely a or b.

EXAMPLE 4 Converting Between Intervals and Inequalities

Convert interval notation to inequality notation or vice versa. Find the endpoints and state whether the interval is bounded, its type, and graph the interval.

(a) $[-6, 3)$ **(b)** $(-\infty, -1)$ **(c)** $-2 \leq x \leq 3$

SOLUTION

(a) The interval $[-6, 3)$ corresponds to $-6 \leq x < 3$ and is bounded and half-open (Figure P.4a). The endpoints are -6 and 3.

(b) The interval $(-\infty, -1)$ corresponds to $x < -1$ and is unbounded and open (Figure P.4b). The only endpoint is -1.

(c) The inequality $-2 \leq x \leq 3$ corresponds to the closed, bounded interval $[-2, 3]$ (Figure P.4c). The endpoints are -2 and 3. **Now try Exercise 29.**

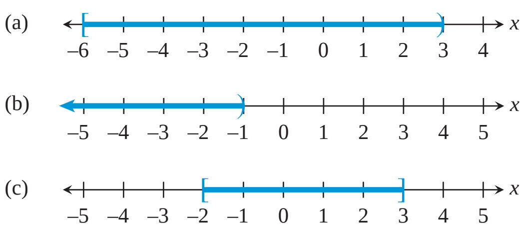


FIGURE P.4 Graphs of the intervals of real numbers in Example 4.

Basic Properties of Algebra

Algebra involves the use of letters and other symbols to represent real numbers. A **variable** is a letter or symbol (for example, x, y, t, θ) that represents an unspecified real number. A **constant** is a letter or symbol (for example, -2, 0, $\sqrt{3}$, π) that represents a specific real number. An **algebraic expression** is a combination of variables and constants involving addition, subtraction, multiplication, division, powers, and roots.

We state some of the properties of the arithmetic operations of addition, subtraction, multiplication, and division, represented by the symbols $+, -, \times$ (or $\cdot$) and $\div$ (or /), respectively. Addition and multiplication are the primary operations. Subtraction and division are defined in terms of addition and multiplication.

Subtraction: $a - b = a + (-b)$

Division: $\frac{a}{b} = a\left(\frac{1}{b}\right), b \neq 0$

In the above definitions, $-b$ is the **additive inverse** or **opposite** of b, and $1/b$ is the **multiplicative inverse** or **reciprocal** of b. Perhaps surprisingly, additive inverses are not always negative numbers. The additive inverse of 5 is the negative number -5. However, the additive inverse of -3 is the positive number 3.

Subtraction vs. Negative Numbers

On many calculators, there are two "$-$" keys, one for subtraction and one for negative numbers or opposites. Be sure you know how to use both keys correctly. Misuse can lead to incorrect results.

The following properties hold for real numbers, variables, and algebraic expressions.

Properties of Algebra

Let u, v, and w be real numbers, variables, or algebraic expressions.

1. Commutative properties
Addition: $u + v = v + u$
Multiplication: $uv = vu$

2. Associative properties
Addition:
$(u + v) + w = u + (v + w)$
Multiplication: $(uv)w = u(vw)$

3. Identity properties
Addition: $u + 0 = u$
Multiplication: $u \cdot 1 = u$

4. Inverse properties
Addition: $u + (-u) = 0$
Multiplication: $u \cdot \frac{1}{u} = 1, u \neq 0$

5. Distributive properties
Multiplication over addition:
$u(v + w) = uv + uw$
$(u + v)w = uw + vw$

Multiplication over subtraction:
$u(v - w) = uv - uw$
$(u - v)w = uw - vw$

The left-hand sides of the equations for the distributive property show the **factored form** of the algebraic expressions, and the right-hand sides show the **expanded form**.

EXAMPLE 5 Using the Distributive Property

(a) Write the expanded form of $(a + 2)x$.
(b) Write the factored form of $3y - by$.

SOLUTION

(a) $(a + 2)x = ax + 2x$
(b) $3y - by = (3 - b)y$

Now try Exercise 37.

Here are some properties of the additive inverse together with examples that help illustrate their meanings.

Properties of the Additive Inverse

Let u and v be real numbers, variables, or algebraic expressions.

Property	Example
1. $-(-u) = u$	$-(-3) = 3$
2. $(-u)v = u(-v) = -(uv)$	$(-4)3 = 4(-3) = -(4 \cdot 3) = -12$
3. $(-u)(-v) = uv$	$(-6)(-7) = 6 \cdot 7 = 42$
4. $(-1)u = -u$	$(-1)5 = -5$
5. $-(u + v) = (-u) + (-v)$	$-(7 + 9) = (-7) + (-9) = -16$